corresponding to 11×11 mesh the second-order upwind scheme requires about the same computer time as the other methods, but its error is much greater. The first-order upwind scheme seems to have larger errors at all meshes.

Altogether it may be concluded that whenever the central difference scheme is stable it requires less computer time to yield a given accuracy. The stability of central differences is increased by the ADI method but at an increased CPU time. The second-order upwind scheme is stable even at $Re = 5000$ (compared with a limit of 1000 for the central differences). It is always somewhat less efficient than central differences. Compared with the first-order upwind scheme it is slightly less efficient at low Reynolds numbers and much more efficient at high Reynolds numbers.

For the impinging jet the typical representative quantity chosen was the maximum vorticity at the downstream boundary of the control volume. This quantity is meaningful only at high Reynolds number, where it should be somewhat lower than the maximum vorticity on the upstream boundary of the control volume. This upper bound was 4.59 in the present calculations.

As in the cavity flow, central differences are most accurate, but in this case the second-order upwind scheme seems to be nearly as good as central differences, and considerably better than the first-order upwind and the exponential schemes, with respect to its accuracy and CPU time demands.

Conclusions

The main conclusion of this study is that the second-order upwind scheme appears to have the potential of yielding sufficient accuracy at an acceptable price (in spite of the fact that it is nonconservative). However, it may become necessary to improve its stability by the application of ADI techniques.

As might have been expected, central difference methods offer the cheapest way to obtain a given accuracy whenever they are stable. The first-order upwind scheme was not usually as accurate as the second-order schemes, but its accuracy was not as bad as might have been expected on the basis of accuracy analysis. There seems to be little point in using the exponential scheme.

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Experimental Evaluation of Sampling Bias in Individual Realization Laser Anemometry

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Nomenclature

N = number of velocity realizations
P = pressure
Re = Reynolds number based on mass average velocity and hydraulic diameter
T = integration time
$\bar{T}_D$ = average Doppler period for N realizations
t = time
U = streamwise velocity
$\bar{U}$ = time-average fluid velocity
U_C = period average velocity
U_B = frequency average velocity
U_i = an individual velocity realization
U_τ = $(\tau_w/\rho)^{1/2}$ = shear velocity
U^+ = average streamwise velocity nondimensionalized with U_τ
y = distance of the center of the probe volume from the wall
y^+ = yU_τ/υ
y_0 = correction to the wall location
b_p = slope of the average velocity profile deduced from pressure gradient measurements
Δ = change in a quantity
$\theta/2$ = beam intersection half-angle
λ = wavelength of laser light
μ = absolute viscosity
υ = kinematic viscosity
ρ = density
τ_w = wall shear stress deduced from pressure gradient measurements
ω_i = weighting function

Introduction

ONE of the principal quantities of interest in turbulent flow measurements is the time-average velocity at a point

$$\bar{U} = \frac{1}{T}\int_t^{t+T} U(t)\,dt \tag{1}$$

This is a straightforward quantity to evaluate experimentally when there is a temporally continuous record of the velocity. However, when the record is discontinuous, as it is when the concentration of scattering particles is dilute and a laser Doppler anemometer is operated in the individual realization or counting mode,[1,2] a precise estimate of the time-average velocity becomes difficult. For an unbiased histogram of random, independent velocity realizations the time-average velocity can be estimated from the ensemble average of the velocity realizations by

$$U_B = \frac{1}{N}\sum_{i=1}^{N} U_i \tag{2}$$

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However, when a dilute concentration of scattering particles is uniformly distributed throughout the fluid, the probability for obtaining a velocity realization with a laser Doppler anemometer is proportional to the instantaneous flow velocity. Consequently, it has been proposed[3] that the sampling of the velocity ensemble is biased toward the higher velocities and that

$$U_B > \bar{U} \tag{3}$$

McLaughlin and Tierderman[3] proposed that a properly weighted ensemble average

$$U_C = \sum_{i=1}^{N} (\omega_i U_i) / \sum_{i=1}^{N} \omega_i \tag{4}$$

will yield a good estimate of the time-average velocity. The proper weighting function, ω_i, was deduced to be proportional to the inverse of the instantaneous velocity vector. However, for most flow situations, it was postulated that a simplified, one-dimensional, weighting function equal to the inverse of the instantaneous streamwise velocity component would be adequate. Since U_i is proportional to the Doppler frequency and $1/U_i$ is proportional to the period of the Doppler signal, Eq. (4) may be rewritten as

$$U_C = (\lambda/2)\sin(\theta/2)/\bar{T}_D \tag{5}$$

where $\bar{T}_D$ is the ensemble average of the period of the individual Doppler signals. Consequently, the one-dimensional corrected estimate is also referred to as the period-average velocity, and U_B is referred to as the frequency-averaged velocity. Karpuk and Tiederman[4] have also proposed that the period-average velocity is the time-average velocity of the fluid at the center of a finite probe volume when the velocity profile is linear.

The objectives of this study were to experimentally demonstrate that sampling bias occurs, and that the one-dimensional correction given by the period average is an adequate estimate of the time-average velocity for non-separating flows.

Experimental Apparatus and Techniques

The viscous sublayer in a fully developed flow of water in a large aspect ratio two-dimensional channel was chosen as the region for the velocity measurements for several reasons. First, since the velocity profile is linear for $y^+ < 6$, it is possible to make accurate estimates of the slope of the velocity profile at the wall from

$$\frac{\Delta \bar{U}}{\Delta y} = \left.\frac{d\bar{U}}{dy}\right)_{y=0} \tag{6}$$

Second, it is also possible to deduce the wall shear stress and the slope of the velocity profile at the wall from measurements of the pressure gradient. Since the turbulence intensity is high in the sublayer region, there is about a 10% difference between the frequency-averaged and the period-averaged estimates of the mean velocity. Consequently, the pressure gradient measurements were used as the correct or standard estimate, and the slopes of the velocity profiles from period averages and frequency averages of the anemometer data were compared to this standard. It should also be mentioned that the narrow size range of the scattering centers plus the design and operation of the laser anemometer were such that the decrease in signal amplitude with increasing velocity discussed by Durst and Whitelaw[5] was not a factor in this study.

Since the viscous sublayer is a thin region of very large velocity fluctuations it was necessary to utilize a laser anemometer with good spatial resolution and a large bandwidth. For these reasons the pedestal canceling optics[6,7] with the probe volume miniaturization described by Karpuk[6] were used in this study. The spatial resolution was enhanced by uniformly bowing the channel walls inward 0.050 in. with the channel running at the desired flow rate. It was then possible to traverse the $\Delta y = 0.00246$ in. thick probe volume up to the channel wall without interference. A dial micrometer coupled to the traverse system allowed the y-locations of the probe volume to be determined to about 0.0001 in. with respect to any other y-location.

All flow measurements were made in the two-dimensional acrylic water channel and stainless steel and plastic (PVC) flow facility described in detail by Reischman[8] and Karpuk.[6] The only modifications made to the channel were the enlargement of the downstream wier tank, and the addition of screens and baffles to redistribute and diffuse the flow exiting the channel. These changes were necessary to damp oscillations in the downstream head that were detrimental to highly accurate pressure drop measurements. The other major modification was the addition of centerline pressure taps at the anemometer test section.[9] The pressure difference between the two pressure taps was measured by a two-fluid micromanometer (water above carbontetrachloride).

The make-up water, normally 250 gallons, was filtered through 0.5 μm filters before passing into the flow loop where it was continuously recirculated. It was then carefully seeded with a measured amount of 5–10 μm sand determined to adequately follow the fluid accelerations.[10] The resulting dilution of scattering centers produced particles in the probe volume less than 4% of the time. All laser anemometer measurements were made 55 channel widths downstream of a sharp-edged Borda-type entrance.

Experimental Results

The three data runs are described in Table 1, including the slope of the mean velocity provile b_p deduced from the pressure drop measurements. The velocity data shown were reduced from magnetic tape using a DISA 55L90 LDA Counter Processor to insure the accuracy of each velocity realization. The biased and corrected estimates of the mean velocity calculated from Eqs. (2 and 5) are shown in columns 8 and 7, respectively. The estimates of the 95% confidence limits for the mean velocity illustrate the high degree of accuracy of the laser anemometer measurements.

Since the accuracy in locating each y-location in the flow with respect to another y-location was greater than the accuracy in locating the channel wall, the wall or $y = 0$ was determined by linear extrapolation from plots of U_B and U_C as functions of y. As shown in Fig. 1, the velocity profiles were very linear, and the amount of this shift in the wall location from a direct determination was relatively small. The values of this shift y_0 are shown in Table 1 and all of the y-locations have been corrected so that $y = 0$ corresponds to the best estimate of the wall location.

The results of the laser anemometer data for run BC-5.2 are compared to the linear profile deduced from simultaneous pressure drop measurements in Fig. 1. The pressure drop profile is represented as a band to illustrate the upper and lower 95% confidence limits for the pressure gradient measurements. This plot is also typical of data runs BC-3 and BC-6. Notice the good agreement between the period average or bias-corrected data and the pressure drop profile. The 95% confidence limits on the LDA data are the same size as the symbol identifying the point.

These three runs are plotted nondimensionally in law of the wall coordinates U^+ and y^+ in Fig. 2. The mean velocity was nondimensionalized with the shear velocity where τ_w was deduced from the pressure drop measurements. Figure 2

Table 1 Experimental results

Run #	Re	b_p (fps in.)	y_0 (in.)	y (in.)	y^+	U_C (fps)	U_B (fps)	% uncertainty in mean	N
BC-3	17,504	27.60 ±8.8%	0.0008	0.00469	2.40	0.1251	0.1430	1.77	1748
				0.00579	2.96	0.1595	0.1802	1.52	2162
				0.00929	4.76	0.2523	0.2865	1.82	1571
				0.01059	5.42	0.2868	0.3254	1.77	1658
BC-5.2	17,959	25.19 ±2.10%	0.0005	0.00719	3.60	0.1728	0.1992	2.00	1465
				0.00824	4.12	0.2028	0.2331	2.27	1114
				0.01104	5.52	0.2739	0.3154	2.11	1310
				0.01214	6.07	0.2970	0.3377	1.99	1332
BC-6	14,838	17.84 ±5.59%	0.0015	0.0100	4.28	0.1774	0.2035	1.95	1479
				0.0111	4.75	0.2016	0.2303	2.03	1323
				0.0129	5.52	0.2311	0.2622	1.96	1350
				0.0139	5.95	0.2446	0.2791	2.23	1084

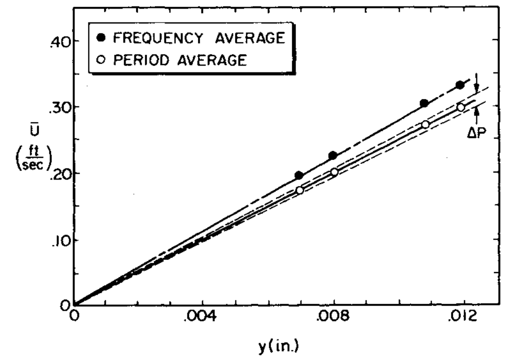

Fig. 1 Comparison of the slope of the frequency-average and the period-average velocity profiles with the pressure gradient.

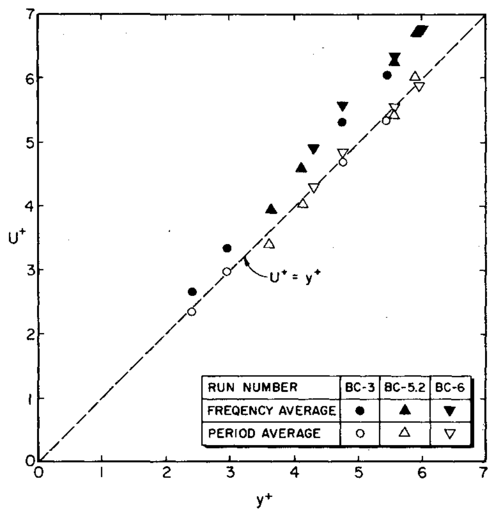

Fig. 2 Nondimensional comparison of the frequency-average and period-average estimates of the mean velocity.

plainly demonstrates the effect of biasing on the mean velocity profile for low duty cycle individual realization LDA measurements. The period average data, corrected for biaisng according to Eq. (5), lie along the $U^+ = y^+$ line, whereas the frequency average (biased) data lie substantially above the $U^+ = y^+$ line. Since $U^+ = y^+$ for the viscous sublayer, Fig. 2 clearly shows that sample biasing occurs and that the proposed one-dimensional correction works well in this case.

Conclusions

The results of this study demonstrate that biased sampling does occur for a dilute concentration of scattering particles uniformly distributed in the fluid. As a result, the ensemble average of individual velocity realizations, the frequency average, is greater than the time-average fluid velocity $\bar{U}$ in turbulent flows. However, in this study a reasonably accurate estimate of $\bar{U}$ is given by the one-dimensional correction where each individual velocity realization in the ensemble is weighted with the inverse of the instantaneous streamwise velocity.

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